On Differences Between SCET and QCDF for $B \to \pi\pi$ Decays

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We give a detailed description of the differences between the factorization and results derived from SCET and QCDF for decays $B \to M_1 M_2$. This serves as a reply to the comment about our work $B \to M_1 M_2$: *Factorization, charming penguins, strong phases, and polarization* [1] made by the authors in [2]. We disagree with their criticisms.

In [1] we derived a factorization formula for exclusive B decays to two light mesons using the soft collinear effective theory (SCET) [3]. Recently, Beneke, Buchalla, Neubert and Sachrajda posted a comment about our work [2], and compared it with their QCDF (QCD factorization) approach [4]. In this paper we compare results and reply to their comments [2].

For easy reference, we summarize a few points made in Ref. [1] that disagree with Ref. [4]. We found that: i) a proper separation of scales $Q^2 \gg E_\pi \Lambda \gg \Lambda^2$ in the factorization theorem are different in SCET and QCDF, and can be regarded as a formal disagreement if desired $(Q = m_b, E_\pi)$; ii) only a subset of $\alpha_s(m_b)$ corrections are currently known, so results for these corrections are formally incomplete; iii) certain amplitudes are sensitive to the treatment of m_c, with parametrically large $\sim v$ contributions from $c\bar{c}$ in the NRQCD region where v is the velocity power counting parameter; iv) current $B \to \pi\pi$ data analyzed at LO in SCET supports values for the parameters $\zeta^{B\pi} \sim \zeta_J^{B\pi}$, in disagreement with numerical inputs adopted in QCDF. We also emphasized that Λ/m_b power corrections need to be of natural size to support model independent phenomenology and verify that the expansion converges, concepts which are sometimes relaxed in QCDF phenomenological analyses. If power corrections change the LO values of $\zeta^{B\pi}$ and $\zeta_J^{B\pi}$ substantially then this would indicate that the power expansion is not converging and a different expansion would be needed if model independent results are desired.

We take this opportunity to also comment on results from Ref. [1] where we found agreement with points made in Ref. [4]. The original starting idea is the same, that factorization theorems for these decays should be derived by making a systematic expansion of QCD in $\Lambda_{\rm QCD}/m_b$, where the terms in this expansion are model independent and unique. Earlier discussion of QCD based factorization methods for nonleptonic decays can be found in [5, 6]. We agree on the scaling in Λ/m_b for the LO amplitudes. There is agreement that input on nonperturbative functions can be obtained from $B \to \pi$ form factors and LO light-cone meson distribution functions $\phi_\pi(x)$ and $\phi_B(k^+)$. We also agree that at LO factorization occurs for amplitudes from light quark penguin loops, as well as tree, and color-suppressed diagrams. Finally, the set of the one-loop hard corrections computed in Ref. [4] determine the Wilson coefficients of the $Q_i^{(0)}$ operators [7].

It is also worth emphasizing that the scope of our two works was different. In Refs. [4, 8] factorization theorems were proposed based on the study of the IR singularities of lowest order diagrams in perturbation theory. Input parameters are taken from QCD sum rules. Certain power suppressed contributions were also included, although the factorization was not extended to this order; as a result, some of these corrections are IR divergent, and cutoffs were used for numerical estimates. In this way the authors of Refs. [4, 8] were able to make predictions for many modes, which however depend on model dependent input. In contrast, in [3] we used operators in SCET to separate the long and short distance physics to all orders in α_s. The decay amplitudes factor, with the long distance physics given by a few universal hadronic parameters. Predictive power was shown to be retained even when $\alpha_s(\sqrt{E\Lambda})$ effects are summed to all orders. We then used data to determine these LO hadronic parameters and obtained a prediction for the heavy to light form factor $f_+(0)$. The factorization theorem also gives a model independent determination of the weak phase γ (or α) using $Br(B \to \pi^0\pi^0)$ as input, but not $C_{\pi^0\pi^0}$ [9].

We organize the remainder of this paper by the four points raised in [2] which we disagree with: 1) that the SCET result [1] is formally equivalent to the QCDF result [4, 8] in all respects, 2) that there is little benefit to avoiding the perturbative expansion at the scale $\mu = \sqrt{\Lambda_{\rm QCD} \, m_b}$, 3) that operators containing a charm quark pair are perturbatively calculable, with corrections suppressed by Λ/m_b regardless of how the scale m_c is treated, and 4) that our phenomenological analysis of recent $B \to \pi\pi$ data, which disfavors certain QCDF input parameters, is flawed because it omits "known" perturbative and power suppressed contributions. A section is devoted to each of these topics.

I. FORMAL COMPARISON

In SCET the separation of scales $Q^2 \gg E_\pi \Lambda \gg \Lambda^2$ can be achieved by matching QCD onto a theory called
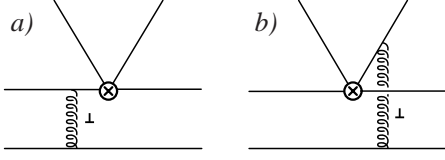
FIG. 1: Examples of QCD diagrams. In Eq. (1) a) and b) both contribute to the $\zeta_J^{B\pi}(z)$ term in the factorization theorem, since the $\perp$-gluon in b) decouples from the upward going pion and can be Fierzed into the other quark bilinear. In Eq. (3) graph a) contributes to the form factor term, and b) contributes to the hard-scattering term.

SCET$_{\rm I}$ to integrate out Q^2 [10], and then matching SCET$_{\rm I}$ onto a final theory SCET$_{\rm II}$ to integrate out the scale $E_\pi\Lambda$ [11]. Performing the first step for $A_{\pi^+\pi^-} = A(\bar{B}\to\pi^+\pi^-)$ at LO in the power counting we found [1]

$$A_{\pi^+\pi^-} = N\bigg\{ f_\pi\int du\,dz\,T_{1J}(u,z)\zeta_J^{B\pi}(z)\phi^\pi(u) \qquad (1)$$
$$+ \zeta^{B\pi} f_\pi\int du\,T_{1\zeta}(u)\phi^\pi(u)\bigg\} + \lambda_c^{(f)}A_{c\bar c}^{\pi\pi},$$

where $N = G_F m_B^2/\sqrt{2}$, the T_i's capture hard $\alpha_s(m_b)$ contributions, and $\zeta_J^{B\pi}$, $\zeta^{B\pi}$ depend on the $\sqrt{E_\pi\Lambda}$ and Λ scales. The analogous expression for the $B\to\pi$ form factor is

$$f_+(0) = T^{(+)}\zeta^{B\pi} + \int dz\,\hat{C}_J^{(+)}(z)\,\zeta_J^{B\pi}(z), \qquad (2)$$

from which we observed that both observables depend on the same universal $\zeta_J^{B\pi}(z)$ and $\zeta^{B\pi}$. Currently $T_{1\zeta}(u)$ is known at $\mathcal{O}(\alpha_s(m_b))$ [4], but $T_{1J}(u)$ is not, hence our statement that the calculation of the hard $\alpha_s(m_b)$ corrections are incomplete. In the notation in [1] the one-loop matching for the Wilson coefficients $b_i^{(f)}(u,z)$ are missing. We do not believe that these facts are disputed in [2]. The amplitude $A_{c\bar c}^{\pi\pi}$ denotes long-distance $c\bar c$ contributions which we take up in a separate section. Short distance $c\bar c$ can contribute to T_{1J} and $T_{1\zeta}$.

In Ref. [4] the QCDF factorization formula was

$$A_{\pi^+\pi^-} = N\,f_+(0)f_\pi\int du\,T_{\rm I}(u)\phi^\pi(u) \qquad (3)$$
$$+ N\,f_\pi^2 f_B\int du\,dx\,dk^+ T_{\rm II}(u,x,k^+)\phi_\pi(u)\phi_\pi(x)\phi_B(k^+).$$

Both of the scales m_b and $\sqrt{E\Lambda}$ are treated perturbatively in $T_{\rm II}$, so this result does not formally distinguish between these scales. This makes it impossible to work to all orders in α_s at the $\mu\sim\sqrt{E\Lambda}$ scale. It is also not possible to sum logarithms between m_b and $\sqrt{E_\pi\Lambda}$ without further factorization of $T_{\rm II}$. Fig. 1 gives an example of how individual diagrams are treated differently in (1) and (3) as explained in the caption. The result in Eq.(1) separates out the hard contributions $\sim Q^2$ regardless of the sensitivity to smaller scales $\sqrt{E_\pi\Lambda}$ and Λ.

Separating the $E_\pi\Lambda \gg \Lambda^2$ scales is more complicated. For $\zeta_J^{B\pi}(z)$ we found

$$\zeta_J^{B\pi}(z) = f_\pi f_B\int dk_+\int dx\,J(z,x,k_+)\phi_\pi(x)\phi_B^+(k_+), \quad (4)$$

where the jet function J starts at $\mathcal{O}(\alpha_s(\sqrt{E_\pi\Lambda}))$ and captures all corrections at this scale. (Multiplicative $\alpha_s(m_b)$ corrections in f_B can be moved into T_{1J} if desired.) The jet function J is now known at one-loop order [12]. As first discussed in [7], using SCET one can write $T_{\rm II}(u,x,k^+) = \int dz[T_{1J}(u,z) - \hat{C}_J^{(+)}(z)]J(z,x,k^+)$; a point mentioned in [2] on which we agree. A full disentangling of the scales $E_\pi\Lambda$ and Λ^2 in $\zeta^{B\pi}$ is still being debated [11, 13, 14, 15, 16]. In SCET$_{\rm I}$ the diagrams that define $\zeta^{B\pi}$ involve the exchange of at least one hard-collinear gluon [11], just as they do for $\zeta_J^{B\pi}$, leading to the expectation that this parameter should also start at $\mathcal{O}(\alpha_s(\sqrt{E_\pi\Lambda}))$, and $\zeta_J^{B\pi}\sim\zeta^{B\pi}$. The parameter counting in QCDF assumes an $\alpha_s(\sqrt{E_\pi\Lambda}))$ only for the hard spectator contributions, and in our notation they have $\zeta_J^{B\pi}\ll\zeta^{B\pi}$. The theoretical issue that blurs the answer to this question is that naively performing similar steps for $\zeta^{B\pi}$ to those giving Eq. (4) give divergent convolution integrals, indicating that this parameter is "non-factorizable" [5, 11, 13, 17]. A complete understanding of the separation of the $\sqrt{E_\pi\Lambda}$ and Λ scales is necessary to sum all logs below the hard collinear scale. For the contribution in Eq. (4) these logarithms have been resumed in Ref. [18], and the running for the other part was studied in [14]. It has been argued [14] that the $\alpha_s(\sqrt{E_\pi\Lambda})$ is absent in $\zeta^{B\pi}$, however this relies on the conjecture that diagrams containing a soft-collinear messenger mode [15] in the theory below the scale $\sqrt{E_\pi\Lambda}$ cancels all endpoint singularities.

II. PERTURBATIONS THEORY IN $\alpha_s(\sqrt{E\Lambda})$

In [1] we showed that an expansion in $\alpha_s(\sqrt{E_\pi\Lambda})$ is not required to obtain predictions from factorization in nonleptonic decays. In other words Eq. (1) has nontrivial implications even without using a perturbative expansion for J in Eq. (4). Since $\sqrt{E_\pi\Lambda}\sim 1.0-1.6$ GeV, we believe that it is useful to consider what predictions can be made by avoiding this expansion.

In our opinion the advantage of using Eq. (4) with an expansion of J in $\alpha_s(\sqrt{E_\pi\Lambda})$ is that it reduces $\zeta_J^{BM}(z)$, which depends on the choice of M, to two more universal nonperturbative functions $\phi_B^+(k^+)$ and $\phi_M(x)$. Thus, the number of unknowns decreases in global analyses involving different light mesons M, and processes like $\gamma^*\gamma\to M$ and $B\to\gamma e\bar\nu$ [19] can be used to provide additional nonperturbative information. This is true both at tree level in $\alpha_s(m_b)$ and at one-loop in $\alpha_s(m_b)$.

There were two main critiques raised in [2]. First, it was argued that there is no point in working to all orders in $\alpha_s(\sqrt{E_\pi\Lambda})$ since perturbation theory in this quantity

is reasonably well behaved, and second that if one works to all orders in $\alpha_s(\sqrt{E_\pi \Lambda})$ then the factorization theorem looses all predictive power beyond tree-level in $\alpha_s(m_b)$.

Regarding the first point, it is always better to use less theoretical assumptions if predictions at the level of precision currently achievable actually do not rely on making this expansion. The perturbative expansion at the scale $\sqrt{m_b \Lambda_{\rm QCD}}$ may in fact converge (recent evidence has been given by the one-loop calculation in [12] and also relations between $B \to D\eta$ and $B \to D\eta'$ decays [20]), however if a prediction is independent of this expansion then there is no need to rely on it. Regarding the second point, it is true that beyond LO in $\alpha_s(m_b)$ the functional dependence of $\zeta_J^{B\pi}(z)$ is required, instead of just the number $\zeta_J^{B\pi} \equiv \int dz \zeta_J^{B\pi}(z)$. However, the same function determines the $B \to \pi$ form factors. Note that when α_s corrections are included in Eq. (3) the functional form of the B meson wave function $\phi_B(k^+)$ is required. So in either case one has moments of one unknown function. We believe that the most important advantage of the $\alpha_s(\sqrt{m_b \Lambda})$ expansion is the universality of the nonperturbative functions mentioned above, rather than the change in how $\alpha_s(m_b)$ corrections are included.

III.   CHARM LOOPS AND $A_{c\bar{c}}$

The size of charm loop contributions to $B \to \pi\pi$ are important. Certain charm loop contributions are from hard ($\sim m_b$) momenta and there is broad agreement [1, 4, 7, 21] that these effects can be computed in perturbation theory. In Eq. (1) they enter in both $T_{1\zeta}$ and T_{1J}. The point being debated is the parametric scaling of non-perturbative contributions from penguin charm quark loops (so-called charming penguins [22]), denoted by $A_{c\bar{c}}^{\pi\pi}$ in Eq. (1).

For the parametric scaling two useful limits are

$$\text{i):} \quad \frac{m_c}{m_b} \ll 1, \qquad \frac{\Lambda}{m_c} \ll 1 \qquad (5)$$
$$\text{ii):} \quad \frac{m_c}{m_b} \sim \mathcal{O}(1), \qquad \frac{\Lambda}{m_c} \ll 1.$$

In [2] this corresponds to the limits $m_b \to \infty$ with m_c fixed and $m_b, m_c \to \infty$ with m_c/m_b fixed respectively, however we believe the description in Eq. (5) makes aspects of the expansion more clear. For example, the charm quark power counting will not be identical to that for light quarks unless $m_c \sim \Lambda$, which is not realized in nature.

In Ref. [1] we focused on nonperturbative contributions from $c\bar{c}$ in the NRQCD region and found that these contributions are only suppressed by v. Note that these $c\bar{c}$'s can still have a total energy $\sim m_b$ as long as their relative velocity $\sim v$ is small.[1] Here v is a place holder

for a nonperturbative matrix element which is parametrically of this size. In charmonium $m_c v \sim 800\,{\rm MeV}$ and $m_c v^2 \sim 400\,{\rm MeV}$ [23], so if we identify one of these scales with $\Lambda_{\rm QCD}$ the v suppression becomes either a $\Lambda_{\rm QCD}/m_c$ or a $\sqrt{\Lambda_{\rm QCD}/m_c}$ suppression. We do not think that the case $m_c v^2 \gg \Lambda_{\rm QCD}$ is physically relevant for charm quarks in QCD. In either case *if we expand* in Λ/m_c these nonperturbative contributions do not enter the LO $B \to \pi\pi$ factorization theorem. However, in practice $v \sim 0.5$ so these corrections are numerically large compared to $\Lambda/m_b \sim 0.1$ power corrections, and can spoil the power expansion.

In Refs. [2, 4, 24] it was argued that the NRQCD $c\bar{c}$ region does not require special treatment due to quark-hadron duality, with smearing from the $q^2 = \bar{x}m_b^2$ of the gluon which the charm annihilate into ($0 \le q^2 \le m_b^2$). Using duality in this sense requires an inclusive hadronic final state, to make it possible for there to be a cancellation of infrared divergences between the virtual and real diagrams to all orders in α_s. For exclusive decays like $B \to \pi\pi$ one must instead prove a factorization theorem to separate hard and infrared contributions. In these proofs one must consider the contributions from all possible momentum regions.

The arguments in [2] assume that the size of the nonperturbative $c\bar{c}$ terms can be estimated based on regions of phase space in q^2, taking limits based on a factorization formula analogous to (3). To the best of our knowledge it has never been proven that the NRQCD $c\bar{c}$ contributions factor in this way. Intuitively we expect that they will not. The gluons whose wavelength is $\sim \Lambda$ do not decouple from the charm pair which are created and annihilated in the octet state. Since the $c\bar{c}$ production and annihilation occur over a distance scale $\sim \Lambda_{\rm QCD}^{-1}$, the soft gluons radiated from energetic quarks produced from the annihilation may not cancel. This can lead to two types of Wilson lines, $Y_n[n \cdot A_{\rm soft}]$ and $Y_{\bar{n}}[\bar{n} \cdot A_{\rm soft}]$ in the soft B-matrix element. In this case the amplitude will involve a new nonperturbative function which has a strong phase from the mechanism found in [25], since the soft function carries information about the final state through n and $\bar{n}$.

Even in the absence of a proof of factorization for $A_{c\bar{c}}^{\pi\pi}$, it should however still be possible to determine its parametric dependence on m_c/m_b, v, and $\Lambda_{\rm QCD}/m_b$ using operators in effective field theories. We find

$$\frac{A_{c\bar{c}}^{\pi\pi}}{A_{LO}^{\pi\pi}} \sim \alpha_s(2m_c)\, f\!\left(\frac{2m_c}{m_b}\right) v\,, \qquad (6)$$

in agreement with [1]. Eq.(6) disagrees with the result in [2] since there is no Λ in the numerator besides that

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[1] Other charm modes besides the ones considered here could also

contribute to $A_{c\bar{c}}^{\pi\pi}$. The consideration of the charm modes above is sufficient to demonstrate the scaling of this nonperturbative contribution.
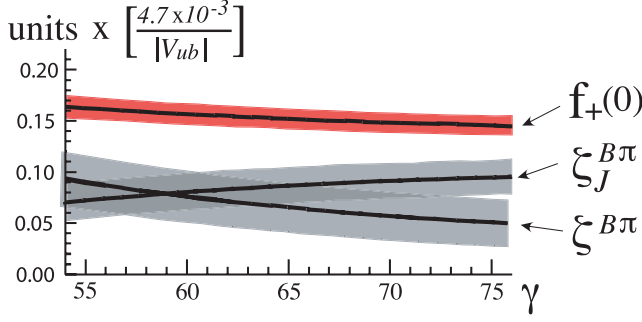
FIG. 2: An update of Fig. 5 in [1] giving model independent results for $\zeta^{B\pi}$, $\zeta_J^{B\pi}$, and the $B \to \pi$ form factor $f_+(q^2=0)$ as a function of γ and $|V_{ub}|$. The shaded bands show the 1-σ errors propagated from the $B \to \pi\pi$ data.

hidden in v. Physically $f(2m_c/m_b)$ encodes the restriction of the charm quarks to be produced with small relative velocity (rather than for example back-to-back with energies $\sim m_b/2$). The factor of v gives the remaining suppression for the charm quarks to be non-perturbative in the NRQCD region. Together these include all "phase-space suppression" factors, which [2] claims were missed in [1]. A derivation of Eq. (6) is given in the Appendix.

IV. NUMERICAL VALUES OF $\zeta_J^{B\pi}$, $\zeta^{B\pi}$

In [1] the parameters ζ and ζ_J were extracted from a LO SCET analysis of the $B \to \pi\pi$ data. New data [26] was presented at ICHEP 2004, and we give here an update of the analysis of these parameters from [1]. We compare our results with the most recent QCDF analysis in [8] and address the criticism in [2].

Assuming isospin symmetry and neglecting the electroweak penguins, the $B \to \pi\pi$ amplitudes can be written as

$$
\begin{aligned}
A(\bar{B}^0 \to \pi^+\pi^-) &= \lambda_u^{(d)} T_c + \lambda_c^{(d)} P, \\
A(\bar{B}^0 \to \pi^0\pi^0) &= \lambda_u^{(d)} T_n - \lambda_c^{(d)} P, \\
\sqrt{2} A(B^- \to \pi^0\pi^-) &= \lambda_u^{(d)} T.
\end{aligned} \tag{7}
$$

This contains five independent hadronic parameters which can be extracted from an isospin analysis. Using the world averages of the data [26] and setting $\gamma = 64°$ one finds the results in the last column of Table I, and

$$
|T_n|/N_\pi = \begin{cases} 0.15 \pm 0.02 \text{(I)}, \\ 0.18 \pm 0.02 \text{(II)} \end{cases}, \tag{8}
$$
$$
P/N_\pi = (-0.024 \pm 0.007) + (0.021 \pm 0.007)i.
$$

Note that our power counting for $A_{c\bar{c}}^{\pi\pi}$ in Eq. (6) gives complex values of a similar size, $|P|/N_\pi \sim \alpha_s(2m_c)vT_c/N_\pi \sim 0.03$, where we took $f(2m_c/m_b) \sim 1$.

Predictions in QCDF involve an expansion at the intermediate scale and use sum-rule calculations for $\phi_B(k^+)$ since this hadronic function is not known from data. Our approach was to instead fit the parameter $\zeta_J^{B\pi}$ to the nonleptonic data and using $\langle x^{-1} \rangle_\pi = 3.0$ which falls within

the range, 3.2 ± 0.4, preferred by fits to the $\gamma^*\gamma \to \pi^0$ data[27]. We also assumed that $\alpha_s(m_b)$ corrections will be of a similar size to neglected power corrections. Numerical justification for this is discussed below.

Factorization formulas like Eq. (1) express the amplitudes T and T_c at leading order in $1/m_b$ in terms of the nonperturbative parameters $\zeta^{B\pi}$ and $\zeta_J^{B\pi}(x)$. Using the leading order SCET relations from [1] we find

$$
\begin{aligned}
\zeta^{B\pi}\big|_{\gamma=64°}^{\text{leading}} &= (0.08 \pm 0.03)\left(\frac{3.9 \times 10^{-3}}{|V_{ub}|}\right), \\
\zeta_J^{B\pi}\big|_{\gamma=64°}^{\text{leading}} &= (0.10 \pm 0.02)\left(\frac{3.9 \times 10^{-3}}{|V_{ub}|}\right).
\end{aligned} \tag{9}
$$

These can be used to predict the $B \to \pi$ form factor at $q^2 = 0$ as $f_+(0) = \zeta^{B\pi} + \zeta_J^{B\pi}$. In Fig. 2 we show how results for our extraction of the ζ's and $f_+(0)$ depend on the input value of γ (with normalization taken using the value of $|V_{ub}|$ preferred by current inclusive fits [28]). Note that the smaller value of $f_+(0)$ favored from our analysis would increase the value of $|V_{ub}|$ from exclusive decays, perhaps bringing it in line with the inclusive analyses.

Working at tree level in the jet function, $f_+(0) = \zeta^{B\pi} + \zeta_J^{B\pi}$ and the ζ_J parameter is given by

$$
\zeta_J^{B\pi} = \frac{\pi\alpha_s(\mu_{\text{int}})C_F}{N_c} \frac{f_B f_\pi}{m_B \lambda_B} \langle y^{-1} \rangle_\pi. \tag{10}
$$

With the input parameters adopted in QCDF [4, 8] at leading order in α_s and Λ/m_b one can find values for the SCET parameters. In the default scenario of [8] we find: $\zeta^{B\pi} = 0.26$, $\zeta_J^{B\pi} = 0.02$, while in their scenario 2: $\zeta^{B\pi} = 0.20$, $\zeta_J^{B\pi} = 0.05$, which are quite different from Eq. (9). There are two possible explanations for this disagreement: i) higher order perturbative and power corrections are important; ii) some of the hadronic input parameters used in QCDF are not supported by the data. The authors of [2] take the first point of view and argue that there are large known perturbative and power corrections to the leading order result.

To compare with QCDF we have calculated the hadronic parameters T, T_c using the analysis in [4, 8]. The results are shown in Table I for two sets of input parameters from [8], their default scenario and their S2 scenario. We have organized the terms according to the expansion advocated in Ref. [1] with $\alpha_s(\sqrt{E\Lambda})$ terms included at LO. The first line in the table shows the LO terms with $\alpha_s(m_b)$ corrections in square brackets, and the second line shows Λ/m_b corrections. We have dropped other Λ/m_b terms which contribute $\lesssim 6 \times 10^{-3}N_\pi$ in the amplitudes T, T_c for default inputs. In contrast to Ref. [8], we evaluate all full theory C_i's at $\mu = m_b$, since below this scale the running does not follow from the usual anomalous dimensions of the electroweak Hamiltonian. Note that $\int dx\phi_\pi(x)/x = 3.3$ in the default scenario and $\int dx\phi_\pi(x)/x = 4.2$ in the S2 scenario. We give the corresponding LO SCET results for these two cases in the table.

	QCDF default scenario	QCDF scenario 2	SCET$_{\text{LO}}$, $\langle x^{-1}\rangle_\pi = \{3.3, 4.2\}$	data
T	$0.354 - [0.037 + 0.019i + \ldots]$ $+(0.006X_H)$	$0.358 - [0.033 + 0.017i + \ldots]$ $+(0.016X_H)$	$1.19\zeta^{B\pi} + \{2.17, 2.44\}\zeta_J^{B\pi}$	$|T| = 0.30 \pm 0.02$
T_c	$0.272 + [0.008 + 0.0i + \ldots]$ $-(0.016 + 0.001X_H + \ldots)$	$0.234 + [0.008 + 0.0i + \ldots]$ $-(0.018 + 0.003X_H + \ldots)$	$0.99\zeta^{B\pi} + \{0.79, 0.74\}\zeta_J^{B\pi}$	$T_c = 0.16 \pm 0.02$

TABLE I: Numerical results for the $B \to \pi\pi$ amplitudes. The first two columns are QCDF results organized so terms with $\alpha_s(\sqrt{E\Lambda})$ are included at lowest order, and using two scenarios for the nonperturbative input parameters from [8]. The numbers in the square brackets are the $\mathcal{O}(\alpha_s(m_b))$ corrections, and the numbers in round brackets are so-called "chirally enhanced" Λ/m_b corrections. Unknown $\alpha_s(m_b)$ and Λ/m_b corrections that contribute at the same order are indicated by ellipses. The third column is the LO SCET result in terms of nonperturbative parameters. The last column is the experimental data from an isospin analysis using $\gamma = 64°$ and $|V_{ub}| = 3.9 \times 10^{-3}$. The results for all amplitudes have a common prefactor $N_\pi = G_F m_B^2 f_\pi/\sqrt{2}$ removed.

From Table I the $\alpha_s(m_b)$ perturbative corrections amount to a $\lesssim 10\%$ shift in the leading order results for T, T_c. The "known" non-perturbative corrections with no X_H factor are $\lesssim 10\%$. Non-perturbative corrections proportional to divergent convolutions in QCDF are $\lesssim 10\%$ for the canonical choice $X_H = 2.4$ from Ref. [8]. All of these are of a similar size to the $10 - 20\%$ corrections we expect from other unknown power corrections. Thus, with the expansion in [1] the model parameters used in QCDF support the claim that the leading order extraction of $\zeta^{B\pi}$ and $\zeta_J^{B\pi}$ from T, T_c is good to $\sim 20\%$. The reason Ref. [2] found that power corrections change $\zeta_J^{B\pi}$ substantially is that their input parameters give a LO result that is small, numerically of a similar size to a typical power correction.

In conclusion, all the perturbative and power corrections which are truly known give rise to small shifts in our LO analysis. The criticisms of Ref. [2] about our work are based on guesses about the size of hadronic parameters and the size of power corrections and thus in our opinion not reliable. Further work is required to get a better understanding of power corrections for non-leptonic B decays. Some interesting work in this direction has been done recently in Ref. [24].

V. CONCLUSIONS

In this letter we discussed the differences between the QCDF approach and SCET approach to factorization in nonleptonic $B \to \pi\pi$ decays, expanding on the points already made in [1] and addressing the criticism in [2]. We also commented on recent SCET work related to these points, which appeared after the publication of [1].

We addressed the main points made in Ref. [2]. Formally, SCET tells us that either there are missing $\alpha_s(m_b)$ corrections in [4, 8] (for $\zeta^{B\pi} \sim \zeta_J^{B\pi}$), or the QCDF counting which treats $\alpha_s(m_b) \sim \alpha_s(\sqrt{E\Lambda})$ relies on $\zeta^{B\pi} \gg \zeta_J^{B\pi}$. Avoiding perturbation theory at the intermediate scale $\mu_{int} \simeq \sqrt{\Lambda_{\text{QCD}}m_b}$ might seem to introduce more nonperturbative $\zeta_J(z)$ functions than expanding in $\alpha_s(\mu_{int})$. However, when restricted to the subset of nonleptonic and semileptonic B decays into pions, this amounts simply to trading one unknown function for another ($\phi_B(k_+)$ vs. $\zeta_J^{B\pi}(z)$). Contrary to the claims made in [2], we still find a complex $A_{c\bar{c}}^{\pi\pi}/A_{LO}^{\pi\pi} \sim \alpha_s(2m_c)v$, indicating that long distance charm penguin contractions can be numerically significant. Finally, we show that our phenomenological analysis of $B \to \pi\pi$ data, and the determination of the two hadronic parameters $\zeta^{B\pi}$ and $\zeta_J^{B\pi}$ remain correct when known perturbative and non-perturbative corrections are estimated as in Ref. [8]. The largest corrections actually come from unknown power suppressed terms, but are still within our error estimate. We leave it to the reader to assess the relative importance of the agreements and disagreements.

APPENDIX A: SCALING OF $A_{c\bar{c}}^{\pi\pi}$

In this appendix we derive the scaling in Eq. (6). First consider limit i). Above the b-mass we have the operator $(\bar{c}\Gamma b)(\bar{d}\Gamma c)$. Next we integrate out the scale m_b and match this operator onto one with massive collinear charm quarks (to ensure they are moving close together, i.e. have invariant mass $\sim 4m_c^2$) and a collinear light d quark,

$$O^{\text{prod}} = \left[\bar{\xi}_{\bar{n},\omega_1}^{(d)}\Gamma'T^A h_v^{(b)}\right]\left[\bar{\xi}_{n'}^{(c)}\Gamma T^A \xi_{n'}^{(c)}\right], \quad (A1)$$

Here $\omega_1 = m_b u$ and the massive collinear charm quarks have $\bar{n}' \cdot p \sim m_b$ and $p_\perp \sim m_c$ with the Lagrangian from [29] (we omit all Wilson lines and other factors that are irrelevant to the power counting). These SCET$_\text{I}$ fields have momenta $p^2 \sim m_b\Lambda$ or m_c^2 which we treat as the same size. The collinear expansion parameters are $\lambda = \sqrt{\Lambda/m_b}$ and $\lambda_c = m_c/m_b$, and $\Gamma = \bar{\not{n}}$ will not contribute, but mass insertions [30] will contribute, such as $\Gamma = \bar{\not{n}}\gamma_\perp m_c/(i\bar{n}\cdot D) \sim \lambda_c$ as seen below. The operator therefore scales as $O^{\text{prod}} \sim m_b^6\lambda^4\lambda_c^3$.

Next we integrate out the scale m_c. The scale $\sqrt{E_\pi \Lambda}$ is close to m_c and can be integrated out at the same time, but the factors generated from doing this are the same as those from the $O_{\text{SCET}_I}^{(0)}$ which occur in the non-charm contributions and so do not effect the relative scaling. Removing m_c in O^{prod} requires matching the charm fields onto NRQCD fields η, χ. There is an operator O_I^{ann} which annihilates the charm in the boosted frame, and at tree level comes with a $1/(4m_c^2)$ prefactor from integrating out a single gluon[2]

$$O_I^{\text{prod}}(0) \; = \; \left[\bar{\xi}_{\bar{n},\omega_1}\Gamma T^A h_v\right]\left[\eta^\dagger T^A(\sigma \cdot L)\chi\right]_{\text{BR}}, \quad \text{(A2)}$$

$$O_I^{\text{ann}}(x) \; = \; \left[\frac{1}{4m_c^2}\chi^\dagger T^A \sigma_\perp \eta\right]_{\text{BR}}\left[\bar{\xi}_{n,\omega_3}\gamma_\perp T^A \xi_{\bar{n},\omega_2}\right].$$

The boost matrix L [31] depends on whether the σ is $\perp$ or longitudinal. The annihilation operator is $\perp$ if $\alpha_s(2m_c)$ is a good expansion parameter, so we take $\sigma_\perp$ in O_I^{prod} in which case $L \sim 1$. Since $[\eta^\dagger T^A \chi] \sim m_c^3 v^3$ this reproduces the m_c^3 for Eq. (A1). Power counting gives

$$O_I^{\text{prod}} \; \sim \; \left(m_b^3 \lambda^4\right)\left(m_c^3 v^3\right),$$

$$O_I^{\text{ann}} \; \sim \; \left(\frac{m_c^3 v^3}{4m_c^2}\right)\left(m_b^3 \lambda^2\right). \quad \text{(A3)}$$

Label momentum conservation in O_I^{prod} implies that the $c\bar{c}$ total momentum has $q^2 \equiv m_b^2 \bar{u} = 4m_c^2$ where $\bar{u} = 1-u$. This gives a delta function in the Wilson coefficient,

$$C_I^{\text{prod}} \simeq \delta\left(\bar{u} - \frac{4m_c^2}{m_b^2}\right). \quad \text{(A4)}$$

The δ-function is expected [24], and is analogous to Eq. (3.13) of [32] for the factorization formula used in production of energetic $c\bar{c}$ state's, whose hadronization is governed by NRQCD. Any δ-function for O_I^{ann} just ensures overall momentum conservation and can be omitted, so we count $C_I^{\text{ann}} \simeq \alpha_s(2m_c)$.

If we now consider the time ordered product capturing the NRQCD region we have

$$C_I^{\text{prod}} C_I^{\text{ann}} \int d^4x \, T\left[O_I^{\text{prod}}(0)O_I^{\text{ann}}(x)\right] \quad \text{(A5)}$$

$$\sim C_I^{\text{prod}} C_I^{\text{ann}}(m_c^{-4}v^{-5})(m_b^3 m_c^3 \lambda^4 v^3)(m_b^3 m_c \lambda^2 v^3)$$

$$\sim (m_b^6 \lambda^6)\, C_I^{\text{prod}} C_I^{\text{ann}}\, v$$

$$\sim \left[O_{\text{SCET}_I}^{(0)}\right]\left\{\alpha_s(2m_c)\delta\left(\bar{u} - \frac{4m_c^2}{m_b^2}\right) v\right\}.$$

Our result for the relative scaling of the $B \to \pi\pi$ amplitudes in limit i), $A_{c\bar{c}}^{\pi\pi}/A_{LO}^{\pi\pi}$, is in curly brackets, and

gives Eq. (6) (once it is integrated over the $\bar{u}$ dependence from the matrix element). Note that the scaling of NRQCD fields together with (A4) account for the "phase space suppression" factors, and that there is an enhancement from the charm propagators being close to their mass shell. We have not been careful to factorize the usoft gluons from the collinear fields etc., so (A5) only indicates the scaling and can not be used to determine the factorization or final operator in SCET$_{II}$. Integrating over a collinear matrix element that depends on $\bar{u}$, $\psi(\bar{u})$, will induce additional dependence on $2m_c/m_b$, so in limit (i) we have to make some assumption about the form of the wavefunction (which we have not proven) to quote a scaling. Taking the form $\psi(\bar{u}) \sim 6\bar{u}(1-\bar{u})$ with $4m_c^2/m_b^2 \simeq 0.44$ unexpanded gives $A_{c\bar{c}}^{\pi\pi}/A_{\pi^+\pi^-} \sim 1.5\, v\, \alpha_s(2m_c)$.

In limit (ii) we can integrate out m_b and m_c simultaneously, and the calculation above of $f(2m_c/m_b)$ will change. Inserting $f(2m_c/m_b) \sim 1$ in Eq. (6) gives $A_{c\bar{c}}^{\pi\pi}/A_{LO}^{\pi\pi} \sim v\, \pi\alpha_s(2m_c)$, which is the result from [1]. In this case there are clearly (u)soft gluons that couple, the $c\bar{c}$, the soft b, and soft spectator.

Note that we have only considered a schematic argument for the scaling of these NRQCD $c\bar{c}$ contributions rather than deriving a factorization theorem, so they are not at the same level of rigor as (1). It is possible that a full consideration of the factorization for $A_{c\bar{c}}^{\pi\pi}$ might uncover an additional subtlety which changes our conclusions. Obviously one can not use the scaling arguments to fix values for $A_{c\bar{c}}^{\pi\pi}$ so we instead used experimental data for the penguin, P. Note that this method of analysis is also appropriate if the power expansion for P does not converge.

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[2] We use BR to denote the fact that the matrix elements of these fields need to be boosted to the B rest frame. The v scaling is assigned from the CM frame, but aside from L is not affected by the boost since there is an invariant that can be formed for the resulting matrix element without kinematic factors.

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